We present in the next example an analysis based on the use of our hypothesis testing summary sheet and an alternative analysis based upon a SAS printout.

Consider the following two-sample problem. Recovery times for two surgical procedures, 1 and 2 are investigated. The data are the observed recovery times and may be considered as independent random samples from two normal populations.

(1) Hypothesis of interest: $H_0: \mu_1 = \mu_2$ vs. $H_A: \mu_1 \neq \mu_2$

Since the sample sizes are small and the validity of the t-test which is appropriate for testing our hypothesis depends on equality of the variances of the two populations, we will need to execute a preliminary test of

(1) Hypothesis’: $H'_0: \sigma_1 = \sigma_2$ vs. $H'_A: \sigma_1 \neq \sigma_2$
and we shall make this test at level $\alpha' = 0.10$.

(2) Decision rule: Reject $H'_0$ if $\tau < F_{n(1)-1,n(2)-1,1-\alpha'/2}$ or if $\tau > F_{n(1)-1,n(2)-1,\alpha'/2}$, where $\tau = \frac{2}{1/s_1^2 - 2}$.

(3) Decision: Since $\tau = 0.848$, $F_{11,8,.05} \sim 3.31$ and $F_{11,8,.95} = 1/2.95 \sim 0.338$ do not reject $H'_0$ and proceed as if the variances were equal.

(2) Decision Rule: Reject $H_0$ if $|\tau| > t_{n(1)+n(2)-2,.025}$, where

$$\tau = \frac{\bar{x} - \bar{y}}{s_p \sqrt{1/n + 1/m}}$$

and

$$s_p^2 = \frac{(n-1)s_1^2 + (m-1)s_2^2}{m+n-2}.$$

(3) Decision: Since $t_{19,.025} = 2.093$ and $|\tau| = 5.7 > 2.093$, reject $H_0$ and conclude that the data does present significant evidence that the mean recovery times for the two procedures differ.

II - Solution using a SAS printout.

The following output of the SAS procedure PROC TTEST can be used to test the above. The output is:

```
Two Sample example                              1
12:34 Monday, August 25, 1997

TTEST PROCEDURE

Variable: RTM

<table>
<thead>
<tr>
<th>T</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Std Error</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>7.80316667</td>
<td>0.73536965</td>
<td>0.21228293</td>
<td>6.78500000</td>
<td>9.25300000</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>9.70886667</td>
<td>0.79778788</td>
<td>0.26592929</td>
<td>8.23180000</td>
<td>10.66200000</td>
</tr>
</tbody>
</table>

Variance: T   DF  Prob>|T|

Unequal -5.6006  16.6  0.0001
Equal -5.6695  19.0  0.0000
```
For H0: Variances are equal, $F' = 1.18$    DF = (8,11)    Prob>F' = 0.7815

and the program that yields this output is

```plaintext
options ls=79;
options pagesize=55;
TITLE 'Two Sample example';
filename rec 'twosam';
DATA rt;
  infile rec;
  INPUT t rtm;
PROC TTEST;
  CLASS t;
  VAR rtm;
```

We begin, as usual, with a statement of the hypothesis and alternative to be tested. The steps that follow are a bit different.

1. Hypothesis of interest : $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$.

   Since the sample sizes are small and the validity of the t-test appropriate for testing our hypothesis depends on equality of the variances of the two populations, we will need to execute a preliminary test of

   (1) Hypothesis' : $H'_0 : \sigma_1 = \sigma_2$ vs. $H'_A : \sigma_1 \neq \sigma_2$

   and will make this test at level $\alpha' = 0.10$.

2. Decision rule: Reject $H'_0$ if the p-value for the variance test is less than 0.10.

3. Decision: Since the p-value for the variance test is .7815, do not reject $H'_0$ and proceed as if the variances were equal.

(2) Decision rule : After failing to reject the hypothesis of equality of variances, reject $H_0$ if the p-value of the t statistic for this data under two-sided hypothesis is less than 0.05.

(3) The appropriate p-value is given on the SAS printout for this case and is less than 0.0000 (it is zero to four places), which is less than 0.05, so reject $H_0$ and conclude that the data does present significant evidence that the mean recovery times for the two procedures differ.
By the way, I generated this data; the x's are N(8,1) and y's are N(9.5,1).